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VIABLE INHOMOGENEOUS MODEL UNIVERSE WITHOUT DARK ENERGY FROM PRIMORDIAL INFLATION

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ABSTRACT

A new model of the observed universe, using solutions to the full Einstein equations, is developed from the hypothesis that our observable universe is an underdense bubble, with an internally inhomogeneous fractal bubble distribution of bound matter systems, in a spatially flat bulk universe. It is argued on the basis of primordial inflation and resulting structure formation, that the clocks of the isotropic observers in average galaxies coincide with clocks defined by the true surfaces of matter homogeneity of the bulk universe, rather than the comoving clocks at average spatial positions in the underdense bubble geometry, which are in voids. This understanding requires a systematic reanalysis of all observed quantities in cosmology. I begin such a reanalysis by giving a model of the average geometry of the universe, which depends on two measured parameters: the present matter density parameter, Ω_m, and the Hubble constant, H_0. The observable universe is not accelerating. Nonetheless, inferred luminosity distances are larger than naïvely expected, in accord with the evidence of distant type Ia supernovae. The predicted age of the universe is 15 ± 1 Gyr. The expansion age is larger than in competing models, and may account for observed structure formation at large redshifts.

Subject headings: Cosmology: theory – Cosmology: large-scale structure of universe — Cosmological parameters — Cosmology: early universe [**arXiv**: gr-qc/0503099]

1. INTRODUCTION

Observations in the past decade have been interpreted as suggesting that 70% of the matter–energy density in the universe at the present epoch is in the form of a smooth vacuum energy, or "dark energy", which does not clump gravitationally. This is supported by two powerful independent lines of observation. Firstly, type Ia supernovae in distant galaxies (Perlmutter *et al.* 1998, 1999; Riess *et al.* 1998, 2004) are dimmer than would be expected in standard Friedmann–Robertson–Walker (FRW) models, especially when it is noted that many independent dynamical estimates of the present clumped mass fraction, Ω_m, suggest values of order 20–30%. Secondly, observations of the power spectrum of primordial anisotropies in the cosmic microwave background radiation (CMBR), most recently by the WMAP satellite (Bennett *et al.* 2003), indicate that the universe appears to be spatially flat on the largest of scales.

A cosmological constant, or alternatively dynamical dark energy, is most commonly invoked to explain the observed cosmological parameters, even though a fundamental origin for such dark energy remains one of the profoundest mysteries of modern physics. However, even in the presence of dark energy at the present epoch, a number of problems remain. One of the most significant problems is that the epoch of reionization measured by WMAP appears at a redshift of order $z \sim 20^{+10}_{-9}$, indicating that the first stars formed much earlier than conventional models of structure formation would suggest. The detection of complex galaxies at relatively large redshifts compounds the conundrum (see, e.g., Cimatti *et al.* 2004, Glazebrook *et al.* 2004).

In recent work, Kolb, Matarrese, Notari and Riotto (2005) have proposed a profoundly different resolution of the "cosmological constant problem". They reason that primordial inflation will have produced density perturbations many times larger than the present horizon volume. We should not view the observable universe as typical of the universe on scales larger than our particle horizon, and the values of cosmological parameters observed should not necessarily be taken as typical of the whole. Furthermore, they suggest that our present observations might be compatible with the observed universe being an underdense bubble in an otherwise spatially flat $k = 0$ FRW universe with an energy density $\Omega_{\rm TOT} = 1$ in ordinary matter. Such assumptions are indeed supported by detailed numerical and analytic calculations made in inflationary models a decade ago (Linde, Linde and Mezhlumian 1996).

The possibility of inhomogeneously defined clock rates is commonly accepted in studies of inhomogeneous cosmologies, such as the Lemaître–Tolman–Bondi (LTB) models. (For a review see Krasiński (1997).) In this *Letter*, I will argue, however, that primordial inflation can give rise to a particular structure in inhomogeneous cosmologies, which allows for a homogeneous cosmic time on the scales of the first bound systems which form, which differs from the "comoving" time parameter of the average late epoch geometry. This novel feature is what distinguishes the present model from previous studies of inhomogeneous cosmologies, as it leads to a new solution of the *fitting problem* (Ellis and Stoeger 1987).

2. OBSERVERS, CLOCKS AND THE FITTING PROBLEM IN INFLATIONARY COSMOLOGY

It is a consequence of the inflationary paradigm that a spectrum of initially small density perturbations is stretched to all observable scales within our past light cone, and also to scales beyond our particle horizon. The fact that this is true for the past light cone is well supported by the CMBR. The hypothesis that such perturbations should extend to super–horizon scales is a feature of most inflationary models, independent of their details.

One important realisation is the fact that since primordial inflation ended at a finite very early time, the scale of super–horizon sized modes, although huge, must have a cut–off at an upper bound. Beyond that scale we assume the universe is described by a spatially flat bulk metric

$$\mathrm{d}s^2_{\mathrm{bulk}} = -\mathrm{d}\tau^2 + \bar{a}^2(\tau)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2), \qquad (1)$$

where $\bar{a}(\tau) = \bar{a}_i(\tau/\tau_i)^{2/[3(1+w)]}$, and we use units with $c = 1$. The precise bulk equation of state $P_{\mathrm{bulk}} = w\rho_{\mathrm{bulk}}$ proves to be inconsequential if $w > -1$, though in accord with the principles of the model we assume only ordinary matter and radiation.

Although the observable universe will undoubtedly be embedded in many regions of under- and over-density, like the smallest figure inside a Russian doll, it is nonetheless reasonable to assume that provided the density perturbation immediately containing our observed universe extends sufficiently beyond our horizon then there is a super-horizon sized underdense bubble containing the observable universe, with matter density *equal to the average matter density we measure*, which we can model as a super–horizon sized underdense region, $\mathcal{S}$.

The matter distribution inside $\mathcal{S}$ is assumed to be *inhomogeneous and "fractal"*, in accord with calculations from primordial inflation (Linde, Linde and Mezhlumian 1996) and in accord with all the evidence of observations of the actual universe, with its large-scale structure of streaming motions of clusters of galaxies, bubbles and voids. Nonetheless, despite this inhomogeneity we do observe an average isotropic Hubble flow. Thus at some level the first step in solving the fitting problem (Ellis and Stoeger 1987) must involve the approximation of the inhomogeneous geometry of $\mathcal{S}$ by an average spacetime which depends on a single cosmic scale factor, viz., the spatially open FRW geometry

$$\mathrm{d}\tilde{s}^2 = -\mathrm{d}t^2 + \tilde{a}^2(t)\left[\frac{\mathrm{d}r^2}{1+r^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right], \quad (2)$$

where parametrically in terms of conformal time, η,

$$\tilde{a} = \frac{a_i\tilde{\Omega}_i}{2(1-\tilde{\Omega}_i)}(\cosh\eta - 1)\,,$$

$$H_i t = \frac{\tilde{\Omega}_i}{2(1-\tilde{\Omega}_i)^{3/2}}(\sinh\eta - \eta)\,, \qquad (3)$$

where $\tilde{\Omega}_i$ is an initial density parameter of the bubble, $\mathcal{S}$, and a_i a constant to be set.

Usually the first step after writing down the metric (2) is to identify comoving observers in this geometry with idealised isotropic observers, namely *observers who measure no dipole anisotropy in the CMBR*. One identifies typical stars in typical galaxies with such observers, apart from the small effects of peculiar velocities. However, even in an homogeneous isotropic FRW model this implicitly assumes a solution of the fitting problem in terms of a complicated matching of asymptotic scales to relate the clocks on geodesics in bound systems, where space is not expanding, to the scale of comoving observers in the Hubble flow, where space is expanding. It is assumed without question that such clocks can be identified.

In general relativity with the particular self–similar inhomogeneous geometry that arises from the evolution of the density perturbations of primordial inflation, the standard solution of the fitting problem is not appropriate, as I argue in detail in a subsequent paper (Wiltshire 2005), henceforth Ref. I. In brief, while the underdense bubble, $\mathcal{S}$, is on its largest scale a single perturbation away from the average bulk density, *due to the scale-invariance of the inflationary spectrum, perturbations on the smaller and intermediate scales that give rise to the first structures nonetheless have the same statistical distribution as the bulk universe, with a mean density distributed about that of the bulk.* The particular structure of inhomogeneity we see today, with its large voids, results in part from a *"particle horizon volume selection bias"* in the initial density perturbations (see Ref. I).

While the metric (2) is still valid for describing the clocks of an observer at an average *spatial* position, in an inhomogeneous underdense bubble such *average spatial positions are in voids and do not coincide with stars and galaxies.* On the other hand, the first bound systems of stars and star clusters which aggregate to typical galaxies, break away from the Hubble flow at early epochs when the average local density of matter in their past light cones is close to the average density of the bulk. Such systems retain fragments of the bulk hypersurface geometry. A clock in a system which has had an approximate stationary Killing vector since the epoch of break away, thus measures the bulk cosmic time parameter, τ, which is "frozen in" from an earlier epoch in the evolution of the universe. Initial perturbations with the selection–biased underdensity of the bubble, $\mathcal{S}$, evolve to form voids.

Given that structure forms in a bottom–up manner, a *"temporal"* Copernican principle operates within $\mathcal{S}$ in the sense that *as average isotropic observers we assume we are located in a bound system formed from those density perturbations which broke away from the Hubble flow when the geometry within their past light cone was indistinguishable from that of the bulk surfaces of homogeneity.* Rather than occupying an average position on a spatial hypersurface, we find ourselves in a bound system that formed from matter which broke from an almost homogeneous Hubble flow at an average epoch. At the present epoch, these same average galaxies, are located in clusters in bubble walls surrounding local voids, in a self–similar hierarchical structure, which we loosely call *fractal.*

Since recent expansion of space occurs primarily in voids, the question of who does or does not measure an isotropic CMBR depends not only on local peculiar velocities but also on whether an observer's line of sight to the surface of last scattering in any direction on the sky averages over the same number and volume distribution of voids and bubble walls, once such structures form. For observers in average galaxies in bubble walls or observers within small voids one would expect a roughly isotropic distribution of bubbles and voids within the self–similar hierarchical matter distribution, and hence an almost isotropic CMBR. Clock rates are determined by local geometry rather than by isotropy, or otherwise, of the CMBR at any location.

To define the local cosmic time, τ, in bound systems in average galaxies, as we have done, in reference to bulk hypersurfaces of matter homogeneity which stretch to regions beyond our present particle horizon may seem puzzling at first. However, prior to inflation, such regions *were in causal contact with the observed universe.* Thus the same basic processes of inflation that lead to isotropy of the CMBR also lead, via structure formation from scale–invariant perturbations, to a definition of inertial frames of observers in average galaxies, namely a new variant of Mach's principle. (See Ref. I for further discussion.)

Since general relativity is a local theory, the average geometry in our first–order solution of the fitting problem is described by (2), (3), but insofar as measurements, including our own, are referred to the frame of average galaxies we must refer them to a coordinates related to those of (2) by a non–trivial lapse function $\gamma(\tau) \equiv \frac{\mathrm{d}t}{\mathrm{d}\tau}$, so that (2) becomes

$$\mathrm{d}\tilde{s}^2 = \gamma^2(\tau)\mathrm{d}s^2\,, \tag{4}$$

where

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + a^2(\tau)\left[\frac{\mathrm{d}r^2}{1+r^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right], \tag{5}$$

and $a \equiv \gamma^{-1}\tilde{a}$. *Since quantities such as the luminosity distance involve null geodesics, for many calculations we can use the conformally related geometry (5).* It is not necessary to solve for the geodesics of observers, defined by local geometry. Only null geodesics probe the vast cosmological scales over which the average geometry (2) is more relevant than other scales in the actual inhomogeneous geometry. Spacelike intervals are never directly measured. Thus by fiat, once we insist on writing the average cosmological geometry in a "synchronous gauge" adapted to the proper time, τ, of the local clocks of observers in average galaxies, then the geometry (5) also comes to define our rods when specifying areas, volumes and densities on the largest scales. The quantity $\gamma(\tau)$ leads to a gravitational time dilation between average galaxies in the gravitational wells of the bubble walls and the ideal comoving clocks in empty voids, as further discussed in Ref I.

To analyse some quantities will require a specification of the entire inhomogeneous fractal geometry within $\mathcal{S}$. This may be possible in the context of a LTB model. However, given the hierarchical nature of the inhomogeneity the LTB mass function would be much more complicated than those of the simple single- or few-void LTB models that have typically been studied to date. Initially, we are only interested in properties defined by the average Hubble flow, although we might expect some modifications of cosmological parameters due to the different range of scales and expansion rates among local voids (Tomita 2001). While solving for intermediate scales, perhaps in terms of a LTB model, will ultimately be necessary, for the purpose of the first approximation in the fitting problem, it suffices to construct a "spherical expansion model" of the underdense region $\mathcal{S}$, in parallel to the well–known "spherical collapse model" (Kolb & Turner 1990).

3. SPHERICAL EXPANSION MODEL

Following the standard approach, we assume the initial density parameter is set sufficiently early that it is very close to unity: $\tilde{\Omega}_i = 1 - \delta_i$, $0 < \delta_i \ll 1$. Furthermore, since the universe has $\tilde{\Omega}_i \simeq 1$ initially we require the comoving scales to match at that epoch. This means we set $\bar{a}_i \simeq a_i$, $\gamma_i \simeq 1$, $t_i \simeq \tau_i$ and $H_i \simeq 2/(3\tau_i)$.

There are various possible Hubble parameters to take into account:
(i) the background Hubble parameter $\bar{H} = 2/[3(1+w)\tau]$ of the metric (1) which cannot be directly measured in $\mathcal{S}$;
(ii) the Hubble parameter of comoving observers in (2), measurable only by unbound observers in average voids

$$\tilde{H}(t) \equiv \frac{1}{\tilde{a}}\frac{\mathrm{d}\tilde{a}}{\mathrm{d}t} \quad ; \tag{6}$$

(iii) the physical Hubble parameter we measure

$$H(\tau) \equiv \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}\tau} \quad ; \tag{7}$$

(iv) an effective (unmeasured) parameter

$$H_{\rm eff}(\tau) \equiv \frac{1}{\tilde{a}}\frac{\mathrm{d}\tilde{a}}{\mathrm{d}\tau} \quad , \tag{8}$$

which is useful for the purposes of calculations.

By our assumptions, while all these parameters coincide at early times, as the universe expands they may differ. As observers in average galaxies, using null geodesics we measure the cosmological geometry (5) referred to global cosmic time, τ, not to the parameter t. We simply need to determine $t(\tau)$ to determine cosmological quantities.

Using Eqs. (2)–(5), the bulk universe scale factor, $\bar{a} = \bar{a}_i\left(\frac{1}{n}\bar{H}_i\tau\right)^n$, where $n = 2/[3(1+w)]$ and $\bar{H}_i = 3nH_i/2$, the density parameter is defined as

$$\tilde{\Omega} = \tilde{\Omega}_i\left(\frac{a_i}{\tilde{a}}\right)^3\left(\frac{\bar{a}}{\bar{a}_i}\right)^{2/n} = \frac{18{H_i}^2(1-\tilde{\Omega}_i)^3\tau^2}{\tilde{\Omega}_i^2(\cosh\eta-1)^3}\,. \tag{9}$$

Comparing this with the standard parametric expression $\tilde{\Omega}(\eta) = 2(\cosh\eta - 1)/\sinh^2\eta$ derived from (2), (3), we obtain a parametric relation for $\tau(\eta)$,

$$H_i\tau = \frac{\tilde{\Omega}_i(\cosh\eta-1)^2}{3(1-\tilde{\Omega}_i)^{3/2}\sinh\eta}\,. \tag{10}$$

The effective parameter (8) is found to be

$$H_{\rm eff}(\eta) = \frac{H_{\rm eff\,0}(1-\tilde{\Omega}_0)^{3/2}(\tilde{\Omega}_0+2)(\cosh\eta+1)^{3/2}}{\tilde{\Omega}_0(\cosh\eta-1)^{3/2}(\cosh\eta+2)} \tag{11}$$

where $\tilde{\Omega}_0 = 2/(1+\cosh\eta_0)$, and a subscript zero refers to the present epoch. The physically measured Hubble parameter (7) is

$$H(\eta) = \frac{(\cosh^2\eta + 2\cosh\eta + 3)H_{\rm eff}(\eta)}{(\cosh\eta+1)(\cosh\eta+2)}\,. \tag{12}$$

and the measured Hubble constant, H_0, is related to $H_{\rm eff\,0}$ by

$$H_0 = \left(\frac{2+\tilde{\Omega}_0^2}{2+\tilde{\Omega}_0}\right)H_{\rm eff\,0}\,. \tag{13}$$

Observe that at early times, $\eta\sim 0$, $H_{\rm eff}\sim\tilde{H}$ and $H\sim\tilde{H}$, as expected but at late times, $H\sim H_{\rm eff}\sim\frac{3}{2}\tilde{H}$. The lapse function is given by

$$\gamma(\eta) = \frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{H_{\rm eff}}{\tilde{H}} = \frac{3(\cosh\eta+1)}{2(\cosh\eta+2)}\,. \tag{14}$$

At the present epoch $\gamma_0 = 3/(2+\tilde{\Omega}_0)$.

We must also be careful to note that the locally measured density parameter, differs from (9) by a volume factor according to $\Omega(\tau) = \gamma^3(\tau)\tilde{\Omega}(\tau)$, so that the present epoch measured matter density fraction is

$$\Omega_m = \frac{27\tilde{\Omega}_0}{(2+\tilde{\Omega}_0)^3}\,, \tag{15}$$

with inverse $\widetilde{\Omega}_0 = 6\Omega_m^{-1/2}\sin\left[\frac{\pi}{6}-\frac{1}{3}\cos^{-1}(\Omega_m^{1/2})\right]-2$.

The expansion age (10) is usefully rewritten as

$$\tau(\eta) = \frac{\widetilde{\Omega}_0(2+\widetilde{\Omega}_0^2)(\cosh\eta-1)^{3/2}}{H_0(1-\widetilde{\Omega}_0)^{3/2}(\widetilde{\Omega}_0+2)^2(\cosh\eta+1)^{1/2}}\,. \qquad (16)$$

The physically measured deceleration parameter $q(\tau) = -H^{-2}\ddot{a}/a = -1-\dot{H}/H^2$, is given by

$$q(\eta) = \frac{7\cosh^2\eta+10\cosh\eta+1}{(\cosh^2\eta+2\cosh\eta+3)^2}\,. \qquad (17)$$

It is equal to the bulk deceleration parameter $\bar{q} = \frac{1}{2}$ at early times $\eta = 0$, but at late times as $\eta\to\infty$, $q\to 0$, so that it is small but positive at the present epoch. This is just as expected from a model without dark energy, and contradicts the claims of cosmic acceleration by Kolb *et al.* (2005). However, we must recall that the supernovae measurements involve luminosity *distances*, and the *interpretation* of cosmic acceleration depends on the time parameter assumed in taking derivatives. The present model may be recognised as mimicking a Milne universe at nearby redshifts, and this still provides a reasonable fit to present SneIa data (Carter *et al.* 2005).

4. OBSERVATIONAL TESTS

To compare with observations Eqs. (12)–(17) need to be expressed in terms of the observed cosmological redshift, z. Great care is needed at this point in identifying physical quantities. It proves simplest to always use the line element (4), (5) remembering that τ corresponds to clock time for observers in average galaxies. It follows that

$$1+z = \frac{a_0}{a} = \frac{\widetilde{a}_0\gamma}{\widetilde{a}\gamma_0} = \frac{(1-\widetilde{\Omega}_0)(2+\widetilde{\Omega}_0)(\cosh\eta+1)}{\widetilde{\Omega}_0(\cosh\eta+2)(\cosh\eta-1)}\,. \qquad (18)$$

The physical solution to eqn. (18) is

$$\cosh\eta = \frac{-1}{2}+\frac{(1-\widetilde{\Omega}_0)(2+\widetilde{\Omega}_0)}{2\widetilde{\Omega}_0(z+1)} + \frac{\sqrt{\widetilde{\Omega}_0 z[9\widetilde{\Omega}_0 z-2\widetilde{\Omega}_0^2+16\widetilde{\Omega}_0+4]+(\widetilde{\Omega}_0^2+2)^2}}{2\widetilde{\Omega}_0(z+1)}\,. \qquad (19)$$

We are now ready for our first cosmological tests. The luminosity distance is readily computed in the standard fashion and is found to be $d_L = (1+z){\gamma_0}^{-1}\widetilde{a}_0\sinh(\eta-\eta_0)$, and since $\sqrt{1-\widetilde{\Omega}_0} = \gamma_0/(\widetilde{a}_0 H_{\mathrm{eff}\,0})$ it follows that

$$H_0 d_L = \frac{(1+z)(2+\widetilde{\Omega}_0^2)}{\widetilde{\Omega}_0(2+\widetilde{\Omega}_0)}\left\{2\cosh\eta-\frac{(2-\widetilde{\Omega}_0)}{\sqrt{1-\widetilde{\Omega}_0}}\sinh\eta\right\}. \qquad (20)$$

As shown by Carter *et al.* (2005) the model gives a good fit to the supernova data, with or without non–baryonic cold dark matter in addition to baryonic matter.

The expansion age is given by substituting (19) in (16). It gives a present day age of the universe of

$$\tau_0 = \frac{2(2+\widetilde{\Omega}_0^2)}{(2+\widetilde{\Omega}_0)^2 H_0}\,. \qquad (21)$$

Using the best–fit value (Carter *et al.* 2005) for $H_0 = 62.7^{+1.1}_{-1.7}\,\mathrm{km\ sec^{-1}\,Mpc^{-1}}$, the age of the universe is $\tau_0 = 14.4^{+0.6}_{-0.5}$ Gyr for $\Omega_m = 0.25\pm0.5$, or alternatively $\tau_0 = 15.2^{+0.7}_{-0.5}$ Gyr for a universe with only baryonic matter, $\Omega_m = 0.075\pm0.055$, using the recalibrated primordial nucleosynthesis bounds of Ref. I.

Importantly, the expansion age is significantly larger at large z: with $H_0 = 62.7^{+1.1}_{-1.7}\,\mathrm{km\ sec^{-1}\,Mpc^{-1}}$, for $\Omega_m = 0.25\pm0.5$ we find $\tau = 4.13^{+0.32}_{-0.26}$ Gyr at $z=2$, $\tau = 1.40^{+0.16}_{-0.13}$ Gyr at $z=6$, and $\tau = 0.29^{+0.05}_{-0.04}$ Gyr at $z=20$. For $\Omega_m = 0.075\pm0.055$ then $\tau = 4.84^{+0.32}_{-0.30}$ Gyr at $z=2$, $\tau = 1.89^{+0.31}_{-0.20}$ Gyr at $z=6$, and $\tau = 0.49^{+0.19}_{-0.09}$ Gyr at $z=20$. This would buy precious time for structure formation early on, and help to explain the redshift of the reionization epoch (Bennett *et al.* 2003), and observations of "early" formation of structure (Cimatti *et al.* 2004, Glazebrook *et al.* 2004).

5. CONCLUSION

As a model cosmology, the present *Fractal Bubble Universe* is remarkable in that its gross features depend only on two already observed parameters, and the results obtained thus far are reasonably promising. If correct, then all observed quantities in cosmology must be systematically reanalysed. The determination of some quantities will require further development of our understanding of the inhomogeneous matter distribution within $\mathcal{S}$, including the largest scale local voids (Tomita 2001), in particular. However, the present model offers a clear framework for calculations, as is demonstrated in Ref. I, where further steps are taken in the reanalysis of the hot Big Bang and observational implications for the CMBR.

It would be ironic that Einstein's idea concerning the irrelevance of the cosmological constant, and his idea of the importance of Mach's principle, may prove to both be right in understanding the universe.

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