Rapidity Gaps and Tests of Models of Diffractive DIS

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Abstract: The strong pseudo-rapidity cuts used in the early analyses of diffractive DIS at HERA restrict the kinematically accessible phase space for a significant region of parameter space in Q^2, β and $x_{\mathbb{P}}$. Here we quantify the regions in which such effects might be observed at HERA and discuss further implications of pseudo-rapidity cuts, including constraints on the exchanged quark virtualities in the diffractive sample, and a source of factorization breaking in the extracted diffractive structure function data. We also include a comparison of models of diffractive dijet production with strong pseudo-rapidity cut data.

1 Introduction

It has been pointed out previously [1, 2] that pseudo-rapidity cuts imposed to select a diffractive Deep-Inelastic Scattering (DIS) sample at HERA lead to a significant reduction in the available phase space for the diffractive interactions for a wide region of parameter space in Q^2, β and $x_{\mathbb{P}}$. Further, the restriction on the phase space equates to a lower limit on the virtuality of the exchanged quark in colour-singlet exchange processes such as that shown in Fig. 1, as well as on the virtuality of the exchanged parton coupling to the pomeron in higher-order diagrams. Such effects should be particularly important in analyses in which very strong pseudo-rapidity cuts are used to select diffractive events [3, 4]. More recent analyses [5], however, are not expected to have significant effects due to pseudo-rapidity cuts, except at relatively large pomeron momentum fractions.

An interesting consequence of the phase space restrictions from pseudo-rapidity cuts is the expected breakdown of factorization of the diffractive structure function into a pomeron flux factor and a pomeron "structure function". Moreover, this is expected to be seen as a relative *reduction* of the diffractive structure function at large values of $x_{\mathbb{P}}$. This is discussed further in Sect. 4.

One may use strong pseudo-rapidity cut data as a further test of models of diffractive interactions. In Sect. 5, we summarize the analyses performed in [2] where models of diffractive dijet production are compared with large pseudo-rapidity gap diffractive structure function data.

In the final section, we summarize the discussion of phase space effects from pseudo-rapidity cuts, and the regions of HERA parameter space in which such effects might be observed.

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[1] Work done in collaboration with G.G. Ross and J. Ellis.

2 Kinematics

The usual variables of standard and diffractive DIS are used throughout: Q^2, x_{Bjorken}, y, M_X^2, β, $x_{I\!P}$ (proton momentum fraction carried by the pomeron), and pseudo-rapidity, η. We neglect the momentum transfer, t, at the proton vertex, as well as quark masses. We define two further variables, k^2 and $p_\perp^2$, which can be parameterized most clearly in the virtual photon–pomeron centre-of-momentum (CMS) system. The first is the virtuality of the exchanged parton that interacts with the pomeron, while $p_\perp^2$ is the component of momentum of the final state parton coupling to the pomeron which is transverse to the virtual photon–pomeron axis. To leading order in α_s, diffractive DIS may be described by the diffractive dijet production diagram of Fig. 1, for which, with CMS scattering angle θ^{cms},

$$k^2 = -\frac{Q^2}{2\beta}(1-\cos\theta^{\mathrm{cms}}), \qquad (1)$$

$$p_\perp^2 = \frac{M_X^2}{4}\sin^2\theta^{\mathrm{cms}}. \qquad (2)$$

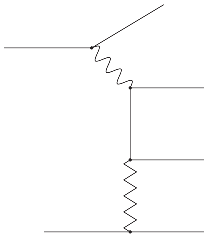

Figure 1: *Diffractive DIS via pomeron exchange.*

In order to relate pseudo-rapidity cuts to phase space parameters at the parton level, one must make some assumption about the hadronization process in the HERA lab frame. As a conservative approximation, we assume that the hadronization radius of a parton (quark or gluon) is about half a unit of pseudo-rapidity. Therefore, in the HERA frame, we assume that

$$\eta_{\max}^{\mathrm{partons}} \approx \eta_{\max}^{\mathrm{hadrons}} - 0.5\,. \qquad (3)$$

3 Consequences of Strong Pseudo-Rapidity Cuts

Pseudo-rapidity gap diffractive events are selected by requiring there to be no activity observed above a low-energy threshold for a region of pseudo-rapidity less than some cut, $\eta_{\max}$, about the forward proton direction. Boosting to the rest frame ($\gamma^* - I\!P$ CMS) of the diffractive system, one finds that a pseudo-rapidity cut, $\eta_{\max}$, in the HERA laboratory (LAB) frame corresponds to a minimum scattering angle in the CMS frame. This means that a pseudo-rapidity cut equates to a cut on the final state phase space from which the diffractive sample is selected[2]. This restriction is expected to be seen as a reduction in the diffractive structure function, $F_2^{D(3)}$, extracted from strong pseudo-rapidity cut data relative to that extracted from data selected with weaker cuts or with a different technique, such as final-state proton detection.

It is clear from Eqs. 1 and 2 that a minimum angle, $\theta_{\rm min}^{\rm cms}$, imposed by event selection cuts implies a minimum exchanged quark virtuality, $k_{\rm min}^2$, and a minimum transverse momentum[3], $p_{\perp\,\rm min}^2$. In Fig. 2 we show the dependence of $k_{\rm min}^2$ on the parameters Q^2, β and $x_{\mathbb{P}}$ for a (strong) pseudo-rapidity cut of $\eta_{\rm max} = 1.8$ corresponding to that used in early H1 analyses [3, 6]. The minimum exchanged quark virtuality, $k_{\rm min}^2$, is a sensitive function of both $x_{\mathbb{P}}$ and β: for large $x_{\mathbb{P}}$, the virtuality constraints are particularly strong, as for small β. By contrast, the diagram of Fig. 3 shows the virtuality constraints for a much weaker pseudo-rapidity cut, of $\eta_{\rm max} = 3.2$, corresponding to that used in more recent H1 and ZEUS analyses. In this case, for most of the range of Q^2, β and $x_{\mathbb{P}}$ used in current HERA analyses, there is no significant phase space reduction imposed by the pseudo-rapidity gap requirement (i.e. no constraint $|k_{\rm min}^2| > 1\,{\rm GeV}^2$).

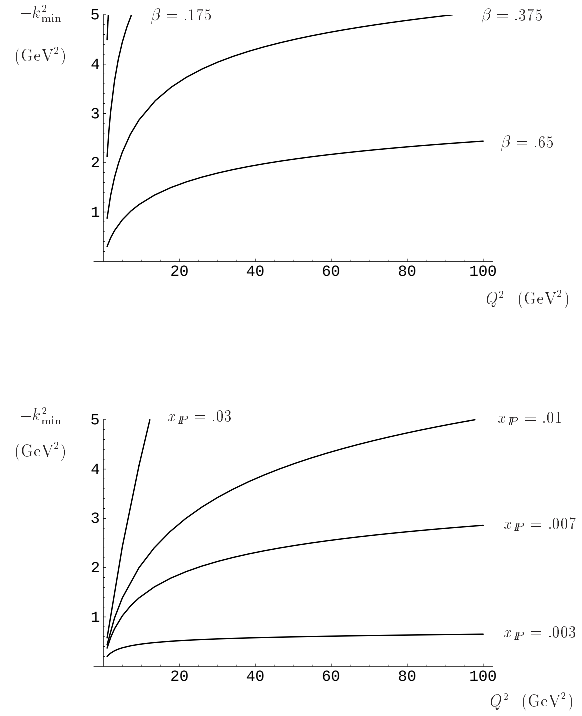

Figure 2: *Dependence of exchanged quark virtuality* $-k_{\rm min}^2$ *on kinematic parameters* Q^2, β, *and* $x_{\mathbb{P}}$. *Both graphs correspond to a pseudo-rapidity cut of* $\eta_{\rm max} = 1.8$, *and for the top graph* $x_{\mathbb{P}} = 0.007$, *while for the lower graph* $\beta = 0.6$.

Since the diffractive cross section falls off rapidly with $p_\perp^2$, and $F_2^{D(3)}$ is extracted by integrating the cross section over the kinematically accessible range of $p_\perp^2$, the diffractive structure

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[2] It should be noted very clearly that for much of the HERA parameter space, for a given pseudo-rapidity cut, this "restriction" does not apply: that is, for a wide range of the parameters, Q^2, β and $x_{\mathbb{P}}$, the pseudo-rapidity cuts *do* still admit contributions from the whole phase space, see, in particular, Fig. 3.

[3] We have checked our results by also considering the rather extreme case that the parent quark is at the edge of the diffractive system nearest to the proton direction, in which case $\theta_{\rm min}^{\rm cms}$ at the quark level is the same as that at hadron level, and still we find significant phase space restrictions for strong pseudo-rapidity cuts [2].

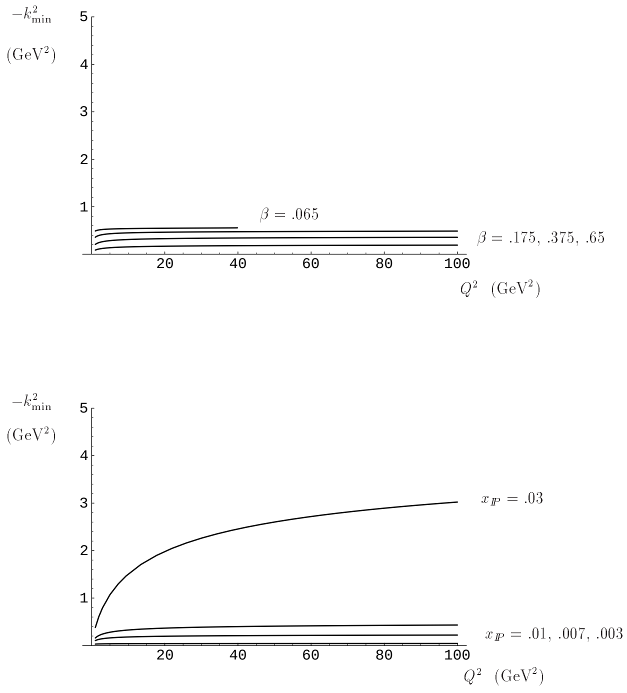

Figure 3: *Dependence of* $-k^2_{\rm min}$ *on kinematic parameters* Q^2*,* β*, and* $x_{I\!P}$ *for a pseudo-rapidity cut of* $\eta_{\rm max} = 3.2$*. For the top graph* $x_{I\!P} = 0.007$*, while for the lower graph* $\beta = 0.6$*.*

function is expected to be very sensitive to pseudo-rapidity cut constraints. Phase space restrictions are expected to lead to a relative reduction in $F_2^{D(3)}$, where the magnitude of this reduction will depend on the parameters Q^2, β and $x_{I\!P}$. That is, at large $x_{I\!P}$ and small β, one would expect the extracted structure function from large pseudo-rapidity gap data to lie below that extracted from data selected with cuts which allow contributions from the whole phase space.

For diffractive DIS events with more than two final state partons, the corresponding exchanged parton virtuality limit is of the order of one third of that for diffractive dijet production [2]. One consequence of this is that at small β one would expect the dijet component of the diffractive events to be strongly cut, thereby enhancing the relative proportion of multi-jet events (i.e. events with three or more diffractively produced final-state partons) in the diffractive sample.

Also, it is clear from Fig. 2 that for a (calculable) wide range of parameter space in Q^2, β and $x_{I\!P}$, strong pseudo-rapidity cuts are selecting only the direct component of the colour-singlet exchange for diagrams corresponding to Fig. 1 (and higher-order diagrams) in which the exchanged parton coupling to the pomeron is far off-shell.

4 Factorization Breaking from Pseudo-Rapidity Gap Cuts

In the hadronic picture of Ingelman and Schlein [7], the diffractive structure function is expected to factorize into an $x_{I\!P}$-dependent pomeron flux factor, $f^{I\!P}(x_{I\!P})$, and an $x_{I\!P}$-independent *pomeron structure function*, $F_2^{I\!P}(Q^2, \beta)$,

$$F_2^{D(3)}(Q^2,\beta,x_{\mathbb{P}}) = f^{\mathbb{P}}(x_{\mathbb{P}})\, F_2^{\mathbb{P}}(Q^2,\beta)\,. \tag{4}$$

However, for data chosen with strong pseudo-rapidity cuts, $p^2_{\perp\,\mathrm{min}}$ is non-zero for a wide range of the kinematic variables Q^2, β and $x_{\mathbb{P}}$, and, in particular, $p^2_{\perp\,\mathrm{min}}$ depends rather strongly on $x_{\mathbb{P}}$. Therefore, even if the full diffractive structure function,

$$F_2^{D(3)} \sim \int_0^{\frac{M_X^2}{4}} \mathrm{d}p_\perp^2\, \frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\,\mathrm{d}\beta\,\mathrm{d}x_{\mathbb{P}}\,\mathrm{d}p_\perp^2}\,, \tag{5}$$

may be factorizable, the diffractive structure function which is extracted with large pseudo-rapidity gap cuts,

$$F_{2\,\mathrm{LRG}}^{D(3)} \sim \int_{p^2_{\perp\,\mathrm{min}}}^{\frac{M_X^2}{4}} \mathrm{d}p_\perp^2\, \frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\,\mathrm{d}\beta\,\mathrm{d}x_{\mathbb{P}}\,\mathrm{d}p_\perp^2}\,, \tag{6}$$

will also depend on $x_{\mathbb{P}}$ through the lower limit of the transverse momentum integral, and hence is not expected to be factorizable.

We can see from Figs. 2 and 3 that k^2_{min} increases strongly with $x_{\mathbb{P}}$, and hence we predict increasingly significant phase space reductions as $x_{\mathbb{P}}$ increases, that is, a greater deviation from factorization in $F_2^{D(3)}$, with a logarithmic plot of $F_2^{D(3)}$ against $x_{\mathbb{P}}$ tending to fall *below* a straight-line fit in the region where k^2_{min} is large.

5 Comparison of Models with Large Rapidity Gap Data

For a given pseudo-rapidity cut, one can calculate the region of parameter space in Q^2, β and $x_{\mathbb{P}}$ in which one would expect to see strong virtuality constraints. One way in which this result can be used to test models of colour-singlet exchange in diffractive DIS is to study a large-k^2 diffractive sample, that is, a diffractive data sample for which the pseudo-rapidity cut and kinematic parameters are chosen such that k^2_{min} is of the order of a few GeV^2. Since k^2_{min} is a sensitive function of β and $x_{\mathbb{P}}$, a comparison with such data would also provide a further study of the (large-k^2) β and $x_{\mathbb{P}}$ structure of the models. There already exists data to test these ideas [6]; the kinematic parameters corresponding to this data for which k^2_{min} is large are shown in [2]. We compared five diffractive dijet production models with diffractive structure function data for which pseudo-rapidity cuts constrained k^2_{min}, and $p^2_{\perp\,\mathrm{min}}$, to be large. Here we summarize those results[4]. The models which we studied are:

1. Donnachie-Landshoff (D-L) form factor model [8] which has a vector-like pomeron in which the virtual quark–pomeron vertex is softened by a form factor $f(k^2) \sim 1/k^2$, which preserves the approximate scale-independence of the diffractive structure function.

2. Ellis-Ross (E-R) form factor model [1] in which a harder form factor, $f^2(k^2) \sim 1/k^2$, is used. This choice leads to a diffractive structure function with only logarithmic dependence on k^2, and $F_2^{D(3)}$ which is therefore relatively insensitive to pseudo-rapidity cuts. This provides

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[4] More details of the models used and the comparison with large pseudo-rapidity gap structure function data are presented in [2].

better agreement with the pseudo-rapidity gap dependence that has been observed [3, 4] than the D-L form factor model.

3. Soft Colour Interaction model [9] in which the dynamics of the diffractive interaction are assumed to be governed by the exchange of one hard gluon, and the pseudo-rapidity gap is protected by subsequent soft gluon exchanges.

4. Scalar Pomeron model of Vermaseren *et al* [10] in which a scalar pomeron couples to quarks with a direct point-like coupling.

5. Two-Gluon model of Diehl [11] in which two non-interacting gluons with non-perturbative propagators are exchanged between the proton and the photon diffractive system.

This analysis proved particularly interesting in the studies of form factor models and of two-gluon exchange: First, in the form factor models, the low-energy cut-off, $\Lambda \sim 1\,\mathrm{GeV}^2$, present in the form factor $f(k^2)$ becomes negligible relative to the k^2-dependence, and the form factors behave simply as $f \sim k^{-2}$ (D-L), and $f \sim k^{-1}$ (E-R). Second, when $p^2_{\perp\,\mathrm{min}}$ is large (which is equivalent to the region of parameter space in which k^2_{min} is large), the two-gluon model [11] exhibits rather different β behaviour from that observed with no k^2_{min} limit.

The comparison of these models with diffractive structure function data is shown in Figs. 4, 5 and 6. In each model the only free parameter is the overall normalization which is determined by a fit to all the data points. The χ^2/dof for each model is shown in Table 1 for two choices of pomeron intercept, $\alpha_{I\!P}(0) = 1.2$ and $\alpha_{I\!P}(0) = 1.08$.

	χ^2/dof	
	$\alpha_{I\!P}(0) = 1.08$	$\alpha_{I\!P}(0) = 1.2$
E-R:	37/41	54/41
D-L:	102/41	97/41
Single gluon:	64/41	57/41
Scalar Pomeron:	48/41	61/41
Two-gluon:	137/41	136/41

Table 1: *Result of fit of various colour-singlet exchange models to the $F_2^{D(3)}$ structure function data from [6].*

As one can see, the E-R form factor approach provides a very good fit to the large rapidity gap diffractive data. Further, the D-L model does not describe the data well, particularly in regions where k^2_{min} is constrained to be very large (small β and large $x_{I\!P}$); here the discrepancy appears to be linked to the very strong k^2-dependence introduced into the cross section by the form factor. Note also that although the $x_{I\!P}$-dependence is introduced into the cross section in each model by hand, the diffractive structure function extracted from the D-L model falls off more rapidly with $x_{I\!P}$ than that from the E-R model. This is a consequence of the sensitivity of the phase space constraints to $x_{I\!P}$ for the data we have considered.

Another interesting result is the failure of the two-gluon model to describe the large rapidity gap data. In particular, the diffractive structure function falls off very rapidly with decreasing β. A likely explanation for this is the dominance of the three jet contribution to diffraction at small β [12]. In terms of this analysis, this effect is expected to be particularly enhanced, as the pseudo-rapidity constraints are weaker for production of three, or more, parton final states. At present we do not know the large k^2_{min} β-dependence of the $q\bar{q}g$ contribution to 2-gluon exchange.

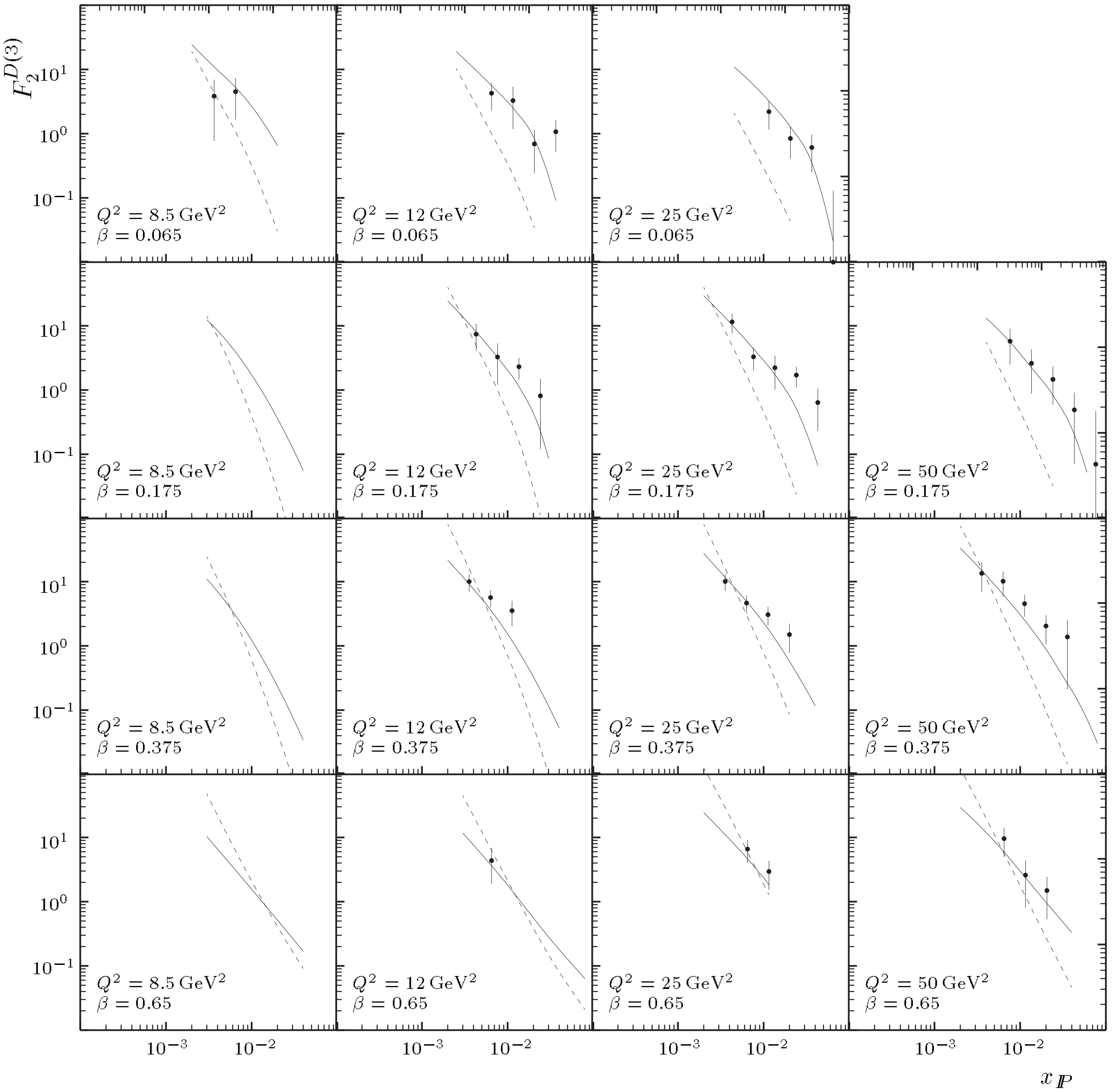

Figure 4: *The E-R and D-L form-factor models are fitted to data for* $F_2^{D(3)}$ *with a virtuality cut. The data sample corresponds to a pseudo-rapidity cut at hadron level of* $\eta_{\max} = 1.8$*, the pomeron intercept is here assumed to be* $\alpha_{I\!P}(0) = 1.08$*, and in the form factors of (D-L) and (E-R) the parameter* Λ *is chosen to be 0.2 GeV. The points correspond to data from [6] for which* $-k^2_{\min} > 1\,\mathrm{GeV}^2$*, and the statistical and systematic errors have been added in quadrature. The solid line is the prediction from the E-R model with the overall normalization determined by a fit to the data points. The dashed line is the D-L model with the same normalization procedure.*

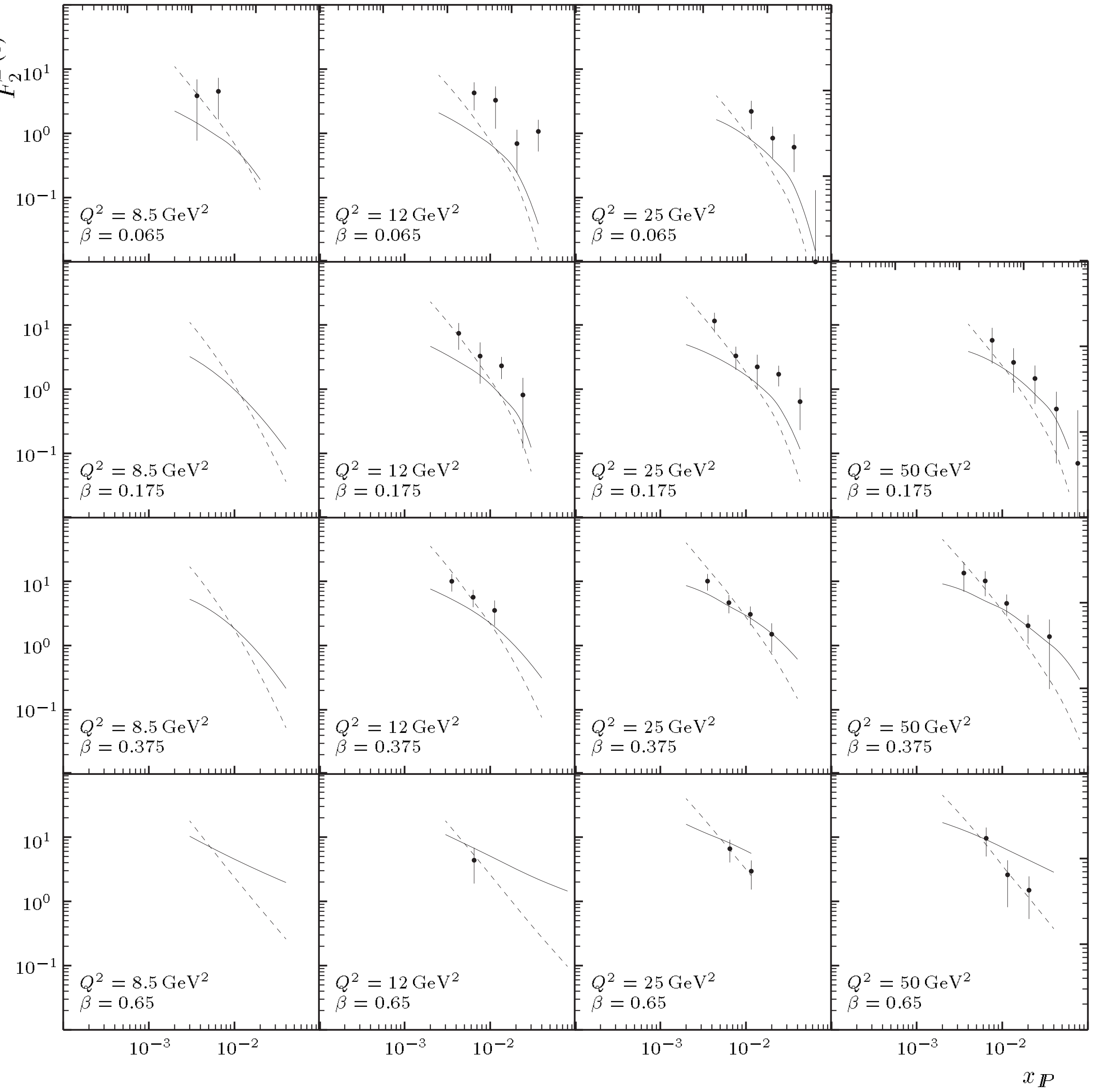

Figure 5: *The single-gluon exchange and scalar-pomeron models are fitted to virtuality-cut data [6] for* $F_2^{D(3)}$*, with parameters as in Fig. 4. The solid line is the prediction from the single-gluon exchange model, and the dashed line is the scalar pomeron model.*

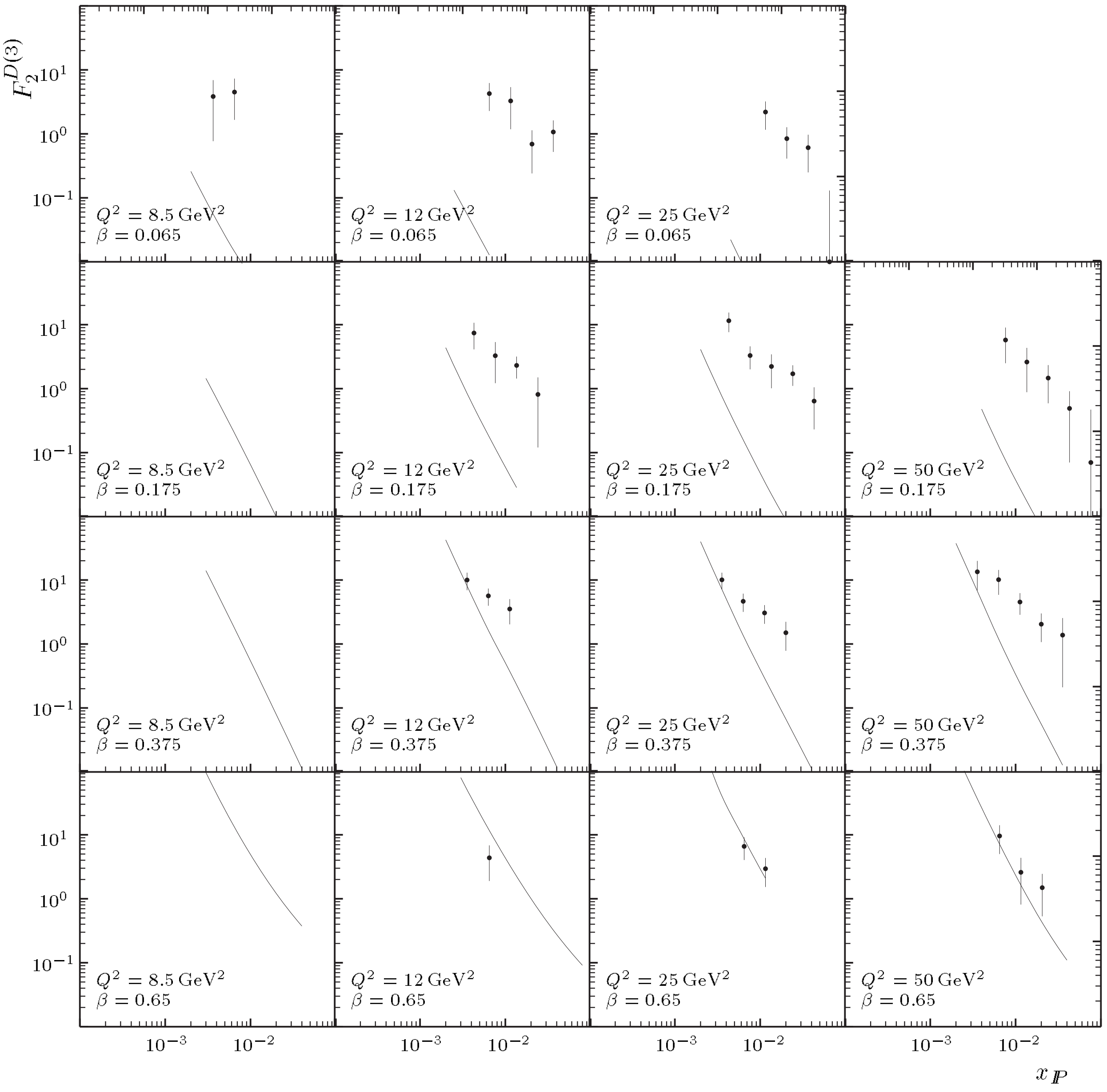

Figure 6: *The two-gluon exchange model is fitted to virtuality cut data [6] for* $F_2^{D(3)}$*, with parameters as in Fig. 4.*

The single-gluon and scalar-pomeron models provide a reasonable fit to the diffractive data. Finally, we find that the strong pseudo-rapidity cut data is not able to distinguish between the two choices of pomeron intercept.

6 Conclusions

If strong pseudo-rapidity cuts are used to select diffractive DIS events, there is a large region of HERA parameter space in which one might see effects such as those describe above due to a reduction in the available phase space for the diffractive scattering interaction. Data for which there are significant constraints on the exchanged quark virtuality can be used as a complementary study of the models of diffractive Deep-Inelastic Scattering. In particular, such cuts select the direct-coupling component of the colour-singlet exchange, and might provide a way to select an enhanced 3-jet contribution to diffraction.

While we have been interested in using strong pseudo-rapidity cut events to explore the large-k^2 structure of diffractive DIS, these results also provide a clear prescription within which to select diffractive events using pseudo-rapidity gap cuts while avoiding the phase space effects we have described. In particular, for relatively weak pseudo-rapidity cuts, one would expect to encounter phase space restrictions only in the region where $x_{\mathbb{P}}$ is large, and β is relatively small.

We compared five diffractive dijet production models with diffractive structure function data for which the pseudo-rapidity cuts and kinematic parameters Q^2, β and $x_{\mathbb{P}}$ lead to a significant restriction on the phase space for diffractive scattering. This has proved to be a rather sensitive test of models of diffractive DIS, for example, the analysis of the two-gluon exchange model in which the β dependence, in particular, differs greatly between the cases of negligible $p^2_{\perp\,\mathrm{min}}$ and large $p^2_{\perp\,\mathrm{min}}$. It would be very interesting to know the large-k^2 structure of the three-jet component of two-gluon exchange to be able to include this contribution in the above analysis. More strong pseudo-rapidity cut data would also be extremely useful in refining the selection between models, and to examine the behaviour of the pomeron intercept as a function of strong pseudo-rapidity cuts.

Acknowledgments

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